Exercise 47

Suppose f and g are continuous functions such that $g(2) = 6$ and $\lim_{x \to 2}[3f(x) + f(x)g(x)] = 36$. Find $f(2)$.

Solution

The fact that f and g are continuous allows us to write

$$\lim_{x \to 2} f(x) = f(2) \quad \text{and} \quad \lim_{x \to 2} g(x) = g(2),$$

which means

$$
\begin{aligned}
36 &= \lim_{x \to 2}[3f(x) + f(x)g(x)] \\
&= \lim_{x \to 2} 3f(x) + \lim_{x \to 2} f(x)g(x) \\
&= 3\left[\lim_{x \to 2} f(x)\right] + \left[\lim_{x \to 2} f(x)\right]\left[\lim_{x \to 2} g(x)\right] \\
&= 3f(2) + f(2)g(2) \\
&= 3f(2) + f(2)(6) \\
&= (3 + 6)f(2) \\
&= 9f(2).
\end{aligned}
$$

Therefore, dividing both sides by 9,

$$f(2) = 4.$$